Simultaneous transits

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Mercury and Venus can be in transit simultaneously across the solar disk. The next two occurrences of such simultaneous transits have been found. We also found several cases when a transit of Mercury or Venus occurs during a solar eclipse.

When an unusual astronomical event occurs, people inevitably ask questions such as 'When will it happen again?' On 2003 May 7 there was a transit of Mercury across the disk of the Sun, and on 2004 June 8 there will be a transit of Venus. Some asked whether it is possible that Mercury and Venus – the two interior planets – can pass *simultaneously* over the solar disk, and when this will happen.

Presently, such simultaneous transits of Mercury and Venus cannot take place, and it is easy to understand why. A transit takes place only when two conditions are satisfied simultaneously:

(1) the planet (Mercury or Venus) must be at *inferior conjunction*, that is, between the Sun and the Earth;

(2) that inferior conjunction must occur close to one of the nodes.

The *nodes* are the two points of intersection of the orbit of the planet with the plane of the Earth's orbit – the ecliptic – and the straight line that connects them is the *line of nodes*. This line passes exactly through the centre of the Sun.

However, the lines of nodes of Mercury and Venus don't coincide. They make an angle of 28° with each other. Consequently, if the two planets are simultaneously in inferior conjunction, they cannot both be near one of their nodes. It is well-known that transits of Mercury can occur only during the first half of May or the first half of November, while transits of Venus are restricted to the first half of June and the first half of December. No overlapping is possible.

But this is the present situation, and things are changing. The good news is that the lines of nodes of Mercury and Venus are slowly approaching each other, making simultaneous transits possible in the far future.

Moving nodes

At the beginning of the year 2000, the longitude of the ascending node of Venus, referred to the mean equinox of the date, was $\Omega_V = 76°.6799$, and it increases by +0°.90112 per century. For Mercury we have $\Omega_M = 48°.3309$, increasing by +1°.18619 /century.

Consequently, the node of Mercury is slowly approaching Venus' node. At the beginning of AD 2000, the difference $\Omega_V - \Omega_M$ was 28°.3490, and this difference decreases by 0°.28507 per century. So we might deduce that the two lines of nodes will coincide 28.3490/0.28507 = 99 centuries in the future, that is, around the year 11,900.

However, the variations of +0.90112 and +1.18619 are not constant, so we must use more accurate formulae. Simon *et.al.*[1] give the following expressions for the longitudes of the ascending nodes referred to the mean equinox of the date:

$$\Omega_M = 48°.33089304 + 42700''.01444t + 63''.14994t^2 + 0''.77259t^3 - 0''.20893t^4 - 0''.00219t^5 + 0''.00016t^6$$

$$\Omega_V = 76°.67992019 + 32437''.57636t + 146''.22586t^2 - 0''.33446t^3 - 0''.23007t^4 - 0''.00088t^5 + 0''.00009t^6$$

where t is the time in Julian *millennia* from the beginning of the year 2000. More precisely, $t = (\text{JD}-2451545)/365250$. Note that, except for the first term of each expression, all constants in the above formulae are in *arcseconds*.

Calculation then shows that the two nodes will coincide for $t = +10.72$, that is, about AD 12,720.

However, the Sun is not a point but its disk has a certain size. Therefore, Mercury and Venus do not need to be exactly at a node for a transit to be possible. Presently, May transits of Mercury are possible when the inferior conjunction occurs up to 2°.7 from the descending node; for November transits, the limit is 4°.8. The difference is due to the large eccentricity of Mercury's orbit. For Venus, the limits are 1°.81 for a June transit, and 1°.72 for a December transit.

So, let us take 4°.8 as the limiting value for a transit of Mercury, and 1°.8 for a Venus transit. The sum is 6°.6. Consequently, simultaneous transits of Mercury and Venus are possible whenever the angle between the lines of nodes of their orbits is smaller than 6°.6. From the above expressions for Ω_M and Ω_V, we deduce that this occurs for $8.1 < t < 13.4$, that is, from AD 10,100 to 15,400, approximately.

Of course, this is only a rough result, because the characteristics of the planetary orbits are slowly changing. The orbits' eccentricities and inclinations vary with time, and the perihelia are moving with respect to the nodes.

Another problem is the accuracy of the expressions for Ω_M and Ω_V over a long time span. Certainly these formulae are not valid for millions of years. Simon[2] mentions that for $t = +10.7$ their accuracy is about 0°.2 or 0°.3. This is sufficient for our purpose, so we may consider the period AD 10,100–15,400 as a good starting approximation.

The question now is: will a simultaneous Mercury–Venus transit actually happen during those 53 centuries? What is the probability of the occurrence of such a remarkable event?

The times of inferior conjunctions of Mercury and Venus do not need to coincide exactly. Central transits can last up to 8 hours. So let us take 8 hours for the limiting value of the time difference between the inferior conjunctions of the two planets. (If the time difference is close to 8 hours, a Mercury transit may end just when a Venus transit begins, or inversely).

How often do inferior conjunctions of Mercury and Venus occur with a time interval of at most 8 hours? Suppose Venus is exactly at inferior conjunction. The synodic period of Mercury is 115.9 days. The favourable time span for Mercury is 16 hours (from 8 hours before to 8 hours after Venus' inferior conjunction); this is the 1/174th part of 115.9 days.

The synodic period of Venus being 583.92 days, there will be about 3315 inferior conjunctions of that planet during the 53 centuries from AD 10,100 to 15,400. For 3315/174 = 19 of them, an inferior conjunction of Mercury will occur at most 8 hours earlier or later. But this is not sufficient to have a simultaneous transit: the conjunctions should occur at less than 1°.8 from Venus' line of nodes. As there are two nodes, and at each node the tolerance is 3°.6 (1°.8 at each side of the node), the expected number of simultaneous transits during the said 53 centuries is $19 \times 3.6/180 = 0.38$, or a probability of 38% to have *one* simultaneous transit.

The actual probability is somewhat less, however, because the two transits may not be nearly central, so one of them may last for much less than 8 hours. Moreover, if the two lines of nodes do not exactly coincide but make with each other an angle of, say, 4° to 6°, even if the two planets are simultaneously in inferior conjunction, one of them may be transiting the solar disk while the other is just missing the Sun. So the probability to have one simultaneous transit during the favourable period from AD 10,100 to 15,400 might be 20% rather than 40.

Numerical integration

At this point, JM asked the help of AV, who performed several accurate calculations by means of a simultaneous numerical integration of the motions of all nine planets, the Moon, and the three large asteroids Ceres, Pallas, and Vesta.[3] The starting conditions were fitted to the DE406 planetary ephemeris of the Jet Propulsion Laboratory.

No simultaneous Mercury–Venus transit between the years 5,000 and 20,000 was found, so the above-mentioned 'favourable' period from about AD 10,100 to 15,400 yields *no* case, as one might have feared. A very close case will occur on 13,425 September 17, when a transit of Mercury will begin about 9 hours after the end of a Venus transit.

JM then found that just *four days* later, on 13425 September 21, Venus will *occult* Mercury! This will happen at 7° west of the Sun. AV confirmed this mutual planetary occultation. (These dates are given in the uniform time scale of Dynamical Time; due to the slowing down of the Earth's rotation, they will probably have shifted to September 13 and September 17, UT, respectively, at least in our Gregorian calendar extrapolated without a change).

Continuing his search to 300,000 years into the future, AV finally found two simultaneous transits: on 69,163 July 26 (but Mercury almost grazing the Sun), and on 224,508 March 27–28 (both planets crossing the middle of the Sun), in the uniform time scale of Dynamical Time. At those epochs, the difference between Dynamical Time and Universal Time will amount to about 133 and 1450 days, respectively, shifting the two events to March 69,163 and April 224,504, Universal Time.

A similar search was performed for the past 280,000 years, but no simultaneous transit was found. A very close case happened on –90,109 February 7 (Julian calendar), when the two transits took place with a time difference of nearly 9 hours. This is too large a difference for the two planets to appear simultaneously on the solar disk. But in this case an error of slightly more than one hour would mean that a simultaneous transit might have occurred for a short time.

For the near-simultaneous event of 13,425, and for the events of 69,163 and 224,508, the instants of least angular distance to the centre of the solar disk, in the uniform time scale of Dynamical Time, and the least distance for a geocentric observer, are given in Table 1.

Several error tests were made and the predictions of the two simultaneous transits of the years 69,163 and 224,508 were confirmed to a good level of confidence. For instance, a second calculation was made by using JPL's earlier DE200 starting conditions instead of DE406. In another test, the actions of the three major asteroids (Ceres, Pallas, Vesta) were removed without readjusting the starting vectors of the planets for optimal fitting to DE406.

AV found a maximum time discrepancy of the order of 5 minutes for the transit of 69,163, and a maximum time discrepancy of the order of half an hour for the transit of 224,508. The discrepancies in the minimum angular distances to the centre of the solar disk were negligible (less than 20 arcseconds).

For the first transit the 'tolerance' in the times (to achieve a simultaneous transit) is about 45 minutes (the time taken by Venus to exit the solar disk after Mercury enters it). For the second transit the tolerance in time is about 2 hours. In both cases the tolerance is substantially higher than the largest error estimates, so it may be concluded that both simultaneous transits will indeed take place.

Table 1. Simultaneous and near-simultaneous transits of Mercury and Venus, years 1 to 300,000

Date	Dynam. time h m	Transiting planet	Distance to Sun's centre (°)
13,425 Sept 17	3 45	Venus	0.074
13,425 Sept 17	19 27	Mercury	0.124
69,163 July 26	16 47	Venus	0.153
69,163 July 26	20 31	Mercury	0.251
224,508 Mar 27	22 59	Venus	0.029
224,508 Mar 28	3 39	Mercury	0.039

The lines of nodes

AV also calculated the times when the lines of nodes of the orbits of Mercury and Venus, *on the ecliptic of the date* (not on the fixed ecliptic of 2000.0) will coincide during the next 450,000 years. He found that, during these 450 millennia, the difference between the longitudes of the ascending nodes, in the sense Mercury minus Venus, is always increasing and

Left: The simultaneous transits of Mercury and Venus across the solar disk on 69163 July 26 (Dynamical Time). Mercury is the small spot at upper left. *Centre:* The solar eclipse during the Mercury transit of 6757 July 5 as seen from a location in the southern hemisphere. *Right:* The solar eclipse during the Venus transit of 15232 April 5 (March 30 or 31, UT date). Celestial (equatorial) North is up in these drawings.

that this occurs at an average rate of +3°.508 per millennium. One complete rotation of one ascending node with respect to the other thus has a mean period of 102,600 years, and the lines of nodes coincide at mean intervals of 51,300 years. These coincidences occur in the following years; A means that the ascending node of Mercury coincides with the *ascending* node of Venus, D with its *descending* node:

12,720 A	67,730 D
118,550 A	171,750 D
224,530 A	274,590 D
325,450 A	375,640 D
425,480 A	

It is near these epochs that simultaneous transits of the two planets are possible. The first epoch, AD 12,720, exactly coincides with what we found by means of the expressions by Simon *et.al.* So these formulae proved to be excellent indeed, at least for the value of the *difference* between the longitudes of the nodes.

For the event of 69,163, Mercury is close to its *descending* node and Venus to its *ascending* node. The inclination of Mercury's orbit on the ecliptic of date is 5°.43, that of Venus 3°.08.

For the event of 224,508, both planets are close to their *ascending* node. The inclination of Mercury's orbit is 8°.93, that of Venus 2°.84. (The present values of the inclinations are 7°.00 for Mercury, 3°.39 for Venus).

Transits and solar eclipses

We investigated another type of dramatic simultaneous event: a transit (of either Mercury or Venus) coinciding with a solar eclipse.

Between the years 1600 and 2300 there are 94 transits of Mercury, and from 2000 BC to AD 4000 there are 81 transits of Venus.[4] At *none* of these transits has there been or will there be a solar eclipse, although for Venus there were the following near misses:

– the total solar eclipse of May 22 of the year –426 (427 BC) ended 9 hours before the beginning of a transit of Venus;
– the total solar eclipse of 1769 June 4 began only 5 hours after the end of the Venus transit of that date.

AV made a search for solar eclipses occurring during a transit, up to AD 16,000, and interesting results were found: nine eclipses during a transit of Mercury, and one during a transit of Venus. Table 2 summarises the results. The second column gives the times of mid-eclipse and of mid-transit in Dynamical Time. To obtain UT, subtract about 17 hours for the event of 6757, and about 138 hours for that of 15,790.

The event of July 8059 was already found in January 1999 by Mr Cees Bassa in the Netherlands, who however failed to find the event of 6757.

Notice how irregularly the events are distributed over time: no case between AD 1600 and 6600, but three hits from 9361 to 9966.

On 5501 June 22 the Moon will occult Mercury during a solar eclipse, but Mercury will then be outside the solar disk.

The largest possible source of error for the data in Table 2 comes from the tidal acceleration of the Moon. According to Chapront (personal communication to JM), its present uncertainty is of the order of 0.01 arcsecs/(century)^2 or less. This means that the quadratically growing error in the Moon's longitude (computed by integration of a model fitted to the JPL DE406 ephemerides) is expected to be about 1 arcsecond after 1000 years and 200 arcseconds after 14,000 years. These correspond to errors in time (*i.e.* the time of mid-eclipse) of the order of 2 seconds of time after 1000 years and 400 seconds (7 minutes) after 14,000 years. Since these uncertainties are much smaller than the time required for a transit of Mercury or Venus, they do not substantially affect the predictions made in Table 2. However, the last event in Table 2 is close to a miss, and if allowance is made for an error three or four times larger than expected, then the event becomes uncertain. Under this hypothesis, the simultaneous transit will take place only if the time error is negative (Moon earlier than predicted) and might instead be a miss if the time error is positive (Moon later than predicted).

Table 2. Solar eclipses during a transit, years 1600 to 16,000

Date	*Dyn. time h m*	*Transiting body*	*Type of eclipse*
6757 July 5	18 15	Moon	partial
6757 July 5	19 06	Mercury	
8059 July 20	10 22	Moon	annular
8059 July 20	11 59	Mercury	
9361 Aug 4	5 52	Moon	annular
9361 Aug 4	6 13	Mercury	
9622 Feb 4	15 19	Moon	annular
9622 Feb 4	16 39	Mercury	
9966 Aug 11	8 13	Moon	total
9966 Aug 11	9 52	Mercury	
10663 Aug 20	1 59	Mercury	
10663 Aug 20	4 05	Moon	total
11268 Aug 25	6 25	Mercury	
11268 Aug 25	8 56	Moon	total
11575 Feb 28	20 05	Moon	annular
11575 Feb 28	22 36	Mercury	
15232 Apr 5	15 38	Venus	
15232 Apr 5	17 42	Moon	total
15790 Apr 20	22 27	Mercury	
15790 Apr 21	1 59	Moon	annular

Other simultaneous transits

After AV had performed the preceding calculations, JM found another rare double event. During the Mercury transit of 2236 November 13–14, there will be a partial *lunar* eclipse. Consequently, an observer on the Moon would see the Earth passing over the Sun during the transit. This was confirmed by using the software *Guide 7.0*.

AV then searched simultaneous transits as seen from Mars, and found that Martians will have to wait until the year 571,741 to see a simultaneous transit of Venus and the Earth (and of course the Moon). A spectacular event indeed!

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Received 2003 June 11; accepted 2003 June 25

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