

## **EXORB, a program for determining orbital elements from observational data.**

### **Release 8.1 for Windows**

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### **Introduction.**

**EXORB8** is an *easy to use, high precision and versatile* orbit determination program working under Windows. The software is an upgrade of EXORB7 (2010). Major new features include the use of extended precision (80 bits) in all calculations, the optional use of a library including the “Big16” perturbing bodies (the 16 more massive main belt asteroids) or even a library including 35 main belt asteroids, the optional modelling and fitting non-G parameters for the Yarkovsky effect, the implementation of an asymmetric model for the non-G accelerations of comets, as well as the modelling and fitting of a strong sudden outburst (single impulse) for comets.

EXORB can be used both to determine an heliocentric orbit (asteroids or comets) and a geocentric orbit (satellites), when a set of observed positions is available. It is based on a full numerical integration of the Solar System, including the nine major planets, the moon and the three asteroids Ceres, Pallas and Vesta [1]. Optionally, the three asteroids can be excluded from the computation, resulting in about 40% increased speed, or more perturbing bodies (16 or 35 total minor planets) can be added. In the case of satellites, also Mercury, Uranus, Neptune and Pluto are excluded from the computation. The program reads the observational data from an ASCII file that must be created by the user with a text editor, using conventions which will be described later, or from a MPC formatted file, or from an AstDys rwo file. Then the program gets from a library file (PL431\*.BIN, according to the option selected for the perturbing bodies) or – optionally - from a custom file the starting conditions which are closer in time to the epoch of the first observation and numerically integrates the Solar System up to this epoch [2]. At this point the program adds a new body to the system, giving to it either trial starting position and velocity, or reading them from a file (STARTING.DAT). Then EXORB starts an iterative fitting procedure, using a Newton-Gauss least-squares algorithm. The time interval covered by the observed data is first integrated using the trial starting conditions, then integrated six more times giving each time a small increment to the corresponding positional or velocity coordinate. For each of the N data point, the differences (residuals) between the observed and calculated positions are recorded and stored in a 7\*N matrix. After the filling of this matrix is complete, use of some matrix algebra [3] produces the vector of the increments to be given to the six starting positional and velocity elements to get an improved set of starting conditions for the body. The procedure is iterated until convergence is reached or the iteration is stopped by the user. The method is suitable both for finding an exact solution from three observations and for finding the least-squares solution from a larger number of observations. The output gives both status vectors at the epoch of the first observation (or integrated up to any other epoch) and classical osculating elements, together with their estimated errors (standard deviations).

### Structure of the input file.

EXORB inputs the observational data from an ASCII file, which should normally (but not necessarily) have an extension .POS. The file can be created with any text editor and even edited from within the program, provided the Windows utility Notepad.exe is accessible from the Solex/Exorb working directory via the default PATH. The input file is automatically generated by the program when converting MPC formatted observations (command [I]) or Astdys-Neodys rwo files (command [\$]), but can also be manually edited, allowing a very flexible kind of input data. If the data are converted from MPC or Astdys format, it is not necessary to bother too much with the complicated options for the structure and format of the POS file, and so go quickly through the following section and keep it only for reference just in case of need. When importing the observations from Neodys rwo files, Exorb does also include the radar data, when present, in the POS file.

The input file can contain up to 16000 data lines and must have the following **format**.

(1) The first three rows must contain some comment text. Additional meaning is given to the last word of the first row, which is interpreted as the name of the body when data are exported from EXORB to SOLEX.

The first non-blank character of the first row is interpreted as a *flag*. If it is a numeric digit other than zero, this tells the program to interpret the time data as a date, otherwise the time will be read as a Julian Day. The date must be written in the format YYYYMMDD.DDDDDDD

(2) The first non-blank character of the second row is interpreted as a *flag*. If it is a positive numeric digit, this tells the program to read **six** numbers from each data line instead of four, which is the default. The last two will be interpreted as the geographic coordinates of the observation site. This becomes a useful possibility when topocentric observations from different sites are available. If it is a negative numeric digit, the program will read **seven** numbers from each data line. The last three will be interpreted as coordinates of the observatory, expressed as Distance from equator plane, Distance from Earth axis (both expressed as a fraction of the equatorial radius), Geographic longitude. This format is automatically adopted when the data are read from a MPC formatted file or from an AstDys rwo file.

(3) The first non-blank character of the third row is interpreted as a numeric *flag* which tells to EXORB how to interpret the data contained in the following lines. If the third row does not begin with a numeric character, the *flag* is set to 0 (zero).

(4) Each of the following lines must contain four numeric data (separated by blanks) referring to an observed position (six numeric data when geographic coordinates of the observation site are included – see previous point 2). The **first** numeric entry is the either the **Julian Day** or a date (YYYYMMDD.DDDDDDD) in all cases. If it is a date, the first non-blank character of the first row must be a numerical digit other than zero. It can either be a Dynamical time TDT (default) or Universal Time UT (in the latter case the appropriate toggle in the Main Menu should be activated).

**A negative epoch (Julian Day or Date) makes the program to skip the corresponding data line.**

**A zero inserted between two lines terminates the input at that point, skipping all the rest.**

Some different formats are valid for the three following positional data, according to the value of the *flag* in row 3:

```
flag is 0:      RA (°°°.°°°°°), Decl (°°.°°°°°), Rho (AU) or
                Lon (°°°.°°°°°), Lat (°°.°°°°°), Rho (AU) or
```

**X (Gm), Y (Gm), Z(Gm)** (orthogonal coordinates)  
*flag is 1:* **RA (hh.hhhhh), Decl (°°.°°°°°), Rho (AU) or**  
**Lon (°°°.°°°°°), Lat (°°.°°°°°), Rho (AU) or**  
**X (AU), Y (AU), Z(AU)** (orthogonal coordinates)  
*flag is 2:* **RA (hh.mmsss), Decl (°°.'''''''), Rho (AU) or**  
**Lon (°°°.'''''''), Lat (°°.'''''''), Rho (AU)**  
**X (AU), Y (AU), Z(AU)** (orthogonal coordinates)

If topocentric observations from multiple sites have to be included, the geographic latitude and longitude (in the format dd.ddd ddd.ddd, positive to East) of each observation site must be added to each line, and the first non-blank character of the second heading line **MUST** be a numeric non-zero digit (see point 2). Of course, if the geographical coordinates are not added to each line, the first non-blank character of the second heading line **MUST NOT** be a numeric character. If only angular information is available, any arbitrary value (e.g. 1.0) can be used for **Rho**.

In all the cases, the coordinates can be geocentric, heliocentric or topocentric. They can be referred to the standard equinox of J2000, to the mean equinox of date, or to the mean fixed equinox of any epoch. They can be geometric positions, or contain light-time effect (but *not* aberration, thus being *astrometric* coordinates), or they can be *apparent* coordinates. **All the option referring to the coordinate system are set from within the Main Menu. The value of the flag only tells to the program in which units the data are written.**

(5) After the last data line the file should contain a 0 (zero)

(6) In the case of topocentric coordinates all from the same observation site (no site informations added to each line), an extra line has to be added *after* the ending zero, containing geographic latitude and longitude of the site (in degrees, West is negative).

## Running ExOrb

**ExOrb8** runs in windowed mode, *not* in full screen mode. After starting the program (either from a CMD prompt or from Windows), the Main Menu appears. Inspect it carefully and, after entering the chosen Input File (some DEMO's are supplied), take care of selecting the correct options through the use of the appropriate toggles, then start the iteration. Some further information about the Main Menu toggles is given in the next section.

**All commands can be given either by mouse clicking on them or by typing the corresponding character. Right-clicking on a command pops up a help window about it.**

Although **ExOrb** often shows impressive performances, it does actually use a brute force approach to the problem of orbit determination. The consequence is that convergence might not be attained, or might be very slow, or might be attained in a wrong local minimum of the merit function (sum of the squares of the residuals). This disease is pretty common, but it can be treated easily and successfully by the four following methods, which can of course be combined.

- The first one can be used when only angular information is supplied by the input file (which is the most common case). In this case a crucial parameter is the starting distance, selected by the toggle **D**, which sets the initial trial value for the position of the body. A good strategy can therefore be to repeat the iteration after changing the value of the starting distance. This can be done without need of returning to the main menu by using the command **R** (Repeat iteration). A few trials made with various

choices of the starting distance should ultimately be successful. For example, if the program is run on the file DEMO.POS by using the default value of 1 AU for the starting distance, convergence occurs at a wrong minimum (with a rather large r.m.s. error). On the other hand, if the starting distance is set at 2 AU, convergence at the correct minimum is reached after a few iterations.

- A second strategy can be effective when the set of observational data is extended over a rather long time interval, covering a large fraction of the body's orbit. In this case convergence can be easier if the set of observations is reduced. This can be done by temporarily inserting a 0 (zero) somewhere in the middle of the file, so that the program reads only the data before the zero. A good choice is to break the file when the body has traveled through an angular path of about 5-10 degrees. When convergence is reached, the program writes the set of optimized starting coordinates and velocities to the file START0.DAT (see later). This can be copied to the file STARTING.DAT (**Note:** this can be done from within EXORB by using the command S in the output menu) and the optimization can be started again toggling *off* the option "Auto Starting Cond." (toggle U), this time using the full set of observational data (removing the intermediate zero). The file STARTING.DAT has the same format of the files \*.SLX used by the program SOLEX (see document SOLEX121.DOC), apart from the fact that it can contain data for only one body. Therefore it can contain either starting positions and velocities or classical osculating elements. The file START0.DAT has the same format, but always contains starting positions and velocities. A corresponding file STARTEL0.DAT is also written, containing classical elements, in the format of **Solex** SLE files.

- A third strategy can be effective when the observations file begins with very few observations, concentrated in a little time span and followed by a large time gap before the next observation appears. This is a situation usually untreatable by the previous two strategies, and needs to temporarily removing the first few observations by adding a minus sign (-) before the time entry of the corresponding line. Then do as above to get the fitting on some or all of the next observations. Once obtained a satisfactory fitting on all the remaining observations, the option "GoToEpoch" can be used to get appropriate starting conditions for the first of the observations which were previously neglected, which can be immediately saved in a new STARTING.DAT file by using the option [S] (Set as Start).

- Finally, a last strategy can be useful when the observations are separated by a rather large angular span. In this case might be convenient to use the option [K] (fit Kepl.), which will cause the fitting to be performed based on classical elements, not on status vectors. This also allows to perform a constrained fitting, using a specific STARTING.DAT elements file and keeping constant one or more of the six classical elements.

## **Main Menu Toggles.**

**All commands and toggles can be entered either by mouse clicking on them or by typing the corresponding character. Right-clicking on a command pops up a help window about it.**

The toggles **O, G, Q, P, M, A, T** tell to the program about the coordinate frame in which the observational data are expressed. Their use is rather obvious, but care must be taken by the user to select the options which are appropriate to any particular data file. Just as an example, it is in the user's responsibility to turn Equatorial Coordinates OFF if the .POS file contains ecliptic positions or to set the Precession to the appropriate equinox if some ancient data are used. **However, when the data are imported from MPC (MPEC format) or Neodys/Astdys rwo files (commands [I] or [S], respectively), Exorb takes care to select the appropriate coordinate frame options automatically.**

- R** Activates the inclusion of relativistic perturbations. It does usually have very little effect.
- S** Must be activated if the orbit of an Earth's satellite is to be computed.
- W** Must be activated to perform a weighted fitting (see later). When activated, clicking on the WF (Weighting factor) number has the effect to uniformly increase or decrease all the uncertainties assigned to the observations in the POS file.
- Z** Should be activated if the time in the .POS file is UT.
- D** Starting distance. Its use has been explained in the previous section.
- U** Must be OFF if the starting conditions have to be read from the file STARTING.DAT
- \*** Monte-Carlo runs. Used for very reliable error estimates (see later).
- Monte-Carlo runs, based on the random exclusion of a random percentage (between 30 and 70%) of observations.
- #** Edit Config File. Calls Notepad to edit the file \CFG\EXORB.CFG, which is read and then rewritten when starting. Useful to change the options which can't be set by the Exorb commands.
- I** Imports an MPC formatted file (without headings and tail lines), converting it to a .POS file. The routine converts MPC time (UTC) to UT1 or TT according to the status of toggle **Z**.
- \$** Imports an AstDys/NeoDys **rwo** file, converting it to a POS file. The routine converts the UTC time to UT1 or TT according to the status of toggle **Z**.
- E** Used to edit the current .POS file. Calls the Windows **Notepad.exe** utility.
- F** Selects the data file.
- X** Allows to constrain the semimajor axis to a specific value when the fitting is made on status vector. If the fitting is done based on Keplerian elements (option [**K**] on), any element can be constrained to a specific value.
- B** Inputs extra massive bodies, whose starting conditions must be given in a file named STARTING.SLX (see the SOLEX manual). This file is best created using SOLEX, going to the appropriate epoch and copying the "FINAL.SLX" generated file to "STARTING.SLX". The option is useful only if the last of the extra massive bodies is going to affect the orbit of the body whose observations are to be fitted.
- C** Custom file. When activated, the starting conditions of the Solar System are read from a custom STARTING.SLF file, rather than from the standard DE431 library. The SLF file (see SOLEX121.DOC manual) must be in the format of status vectors, not classical elements. It is convenient although not necessary, to have the starting conditions of the file STARTING.SLF set for the same epoch as the first observation of the input POS file.
- L** Switches between different libraries of perturbing bodies (the same used by Sorex12), the basic one containing only planets from Mercury to Pluto, the larger one containing, in addition, 35 asteroids. The B16 library contains the 16 heaviest asteroids and corresponds to the dynamical model most frequently used by JPL.
- V** Adaptive Stepsize. **To be turned ON in the case of bodies approaching a planet or approaching the Sun within the orbit of Venus.**
- K** If activated, the fitting is performed based on variation of the classical elements, not of the status vector.
- +** Fit Extra Body. Allows to fit the Keplerian elements of the extra body initially given in the file STARTING.SLX.
- ^** Special command allowing to fit the components of a single impulse acting at a specific time. Not a concern unless for a very specialized user. See the on-line help for details.
- <** Special command allowing to implement and fit an asymmetric model for the non-gravitational forces acting on a comet. Not a concern unless for a very specialized user. See the on-line help for details.

- 1-6** Select which parameters will undergo optimization. Left-click switches the parameter ON/OFF, double click allows to change the starting value of the parameter, right-click allows to edit (via call to Notepad.exe) the whole STARTING.DAT file. Note that upon double-click a warning message is issued if the data in the STARTING.DAT file do not match the kind of parameters that are to be fitted, for example if you are trying to edit a coordinate while the fitting is based on Keplerian elements (or vice-versa).
- 7-@** Allow to select which parameters will undergo optimization.  $A1$ ,  $A2$  and  $A3$  are comet's non gravitational parameters, which can be meaningful to evaluate and/or optimize only when either the positional observations are extremely good (which is not usually the case) or they include *at least* two successive orbits of a periodic comet.  $AF$  is a scale factor (0.111262 according to Marsden) which multiplies  $A1$ ,  $A2$  and  $A3$ . If the latter are taken from the literature, it is optionally possible to retain their values and to optimize only the scale factor  $AF$ . In the case of Earth's satellite, the parameters 7-0 become  $Qd$ ,  $QE2$ ,  $QE3$ ,  $QEX$ .  $Qd$  is a drag empiric coefficient,  $QE2$  is proportional to the Oblate Earth's  $J2$  coefficient,  $QE3$  is proportional to the Oblate Earth's  $J3$  coefficient,  $QEX$  is not used (free for future implementations). If both  $QE2$  and  $QE3$  are set to zero in the STARTING.DAT file, the program assigns default initial values to them. It looks rather weird to optimize the  $QE2$  and  $QE3$  parameters, which should be constant for all satellites, being dependent on nothing else but the Earth's figure. Nevertheless, on a **purely empirical basis**, the optimization of these parameters results in a better fitting, when the ECI positions computed by the TRAKSTAR program [5] are used as observational data (see SOLEX 9 documentation for a description of the procedure).
- If the option [+]  
(Fit Extra Body) is selected, the parameters become the Keplerian elements of the body in the file STARTING.SLX, which in this case must contain only one body. See the **warning** to the last option **J**.
- Y** Toggles between Marsden non-gravitational model for comets and an analogous model for Yarkovsky accelerations for asteroids.
- J** To be used in conjunction with the **B** option. Optimizes the mass of the last of the extra massive bodies along with the orbital elements of the observed test body. **Warning**: the optimization can only be successful if the mass of the extra body **does** have a substantial perturbing effect on the test body. Otherwise, the optimization might not converge or might give unreliable results.

### Output.

The screen output of EXORB8 is self-explaining. The output elements are referred to the standard equator and equinox of J2000. The parameter  $\Sigma$  is the root mean square angular deviation of the observed from the calculated data. When convergence is reached or the iteration is stopped by the user, the estimated standard deviations of the optimized elements are given. The parameter  $Rj0$  is involved in a relativistic correction [1], which is related to the semimajor axis of the orbit ( $Rj0 = 9GM/ac^2$ ). Relativistic perturbations have usually negligible effect; anyhow they can be toggled on and off in the main menu by the toggle **R**. In the output menu, the command **R** does instead display the largest *residuals* (differences between observed and calculated values) in decreasing order, which can help in deciding whether any of the data point is bad and ought to be removed (temporary removing of a data point can be effected by putting a minus sign in front of the time, which is the first entry in the corresponding line of the input file). Automatic rejection of the observations giving residuals above a defined threshold can be done with the command **B**, and the switching between an absolute threshold in arcsecs or a relative threshold with respect to the assumed rms of the observation can be done by editing the configuration file (there is no specific command in the menu for this choice). The

full set of residuals is written to the file RESID.DAT. The first entry of each line of residuals is the corresponding line number of the position's file, no matter whether or not some observations have been excluded from the optimization. This makes much easier to identify in the positions' file which observation corresponds to any particular residual. The command **S** writes the starting conditions resulting from the fitting to the file STARTING.DAT, (and to the file STARTING.SLX if fitting of the Extra body was activated) in view of further iterations on an extended set of data.

After the iteration is stopped, if output is required for an epoch different from that of the first data point, this can be obtained by the command **G** (GoTo Epoch). In this case the time interval from the starting to the target epoch is integrated twice (the second time using slightly different starting conditions), to allow an estimate of the error propagation over the time interval.

The command **K** (Make clones) creates the file CLONES.SLX containing the specified number of clones of the nominal solution, generated by a procedure making use of the covariance matrix. This file is an alternative to **RANDOM.SLX** (generated by the time-consuming MonteCarlo routine), and both are very useful in estimating the error propagation with time, after propagating the orbits of the set of clones with **Solex**.

The command **X** (Export SLX) exports the current solution START0.DAT (or START1.DAT when a GO to Epoch command was given) to a SLX file named as the current POS file with an "X" character appended to the name.

#### Files written by Exorb81.

The following is an alphabetic list of the files that are (or might be) written by Exorb81, with a short explanation.

<b>BESTELE.SLX</b>	Best intermediate solution in the whole sequence of iterations, in form of status vector. Useful as a new starting point in the case of failure to converge.
<b>BESTPOS.SLX</b>	Best intermediate solution in the whole sequence of iterations, in form of orbital elements. Useful as a new starting point in the case of failure to converge.
<b>CALCD.DAT</b>	Calculated positions of the body, written at the end of the fitting.
<b>CLASSEL.LOG</b>	Table of keplerian elements of the set of clones generated by the MonteCarlo procedure.
<b>CLASSELX.LOG</b>	Written only if the elements of an external perturbing body have been optimized: Table of keplerian elements of the set of clones generated by the MonteCarlo procedure.
<b>CLONES.SLX</b>	File of clones of the nominal solution generated by the command <b>K</b> to be imported by <b>Solex</b> .
<b>COVAREXT.DAT</b>	Matrix of covariances in high numerical precision, to be used for generating clones.
<b>COVMAT.DAT</b>	Matrix of covariances in low numerical precision, displayed by the command <b>R</b> .
<b>EXORB.CFG</b>	Configuration file, storing the latest settings and read at start.
<b>FIRST0.SLF</b>	Full SLF status vectors file, written at the epoch of the first observation.
<b>FIRST0.SLX</b>	SLX status vector file of the optimized body, written at the epoch of the first observation.
<b>IMPULSE.LOG</b>	Log file of the impulses in a set of clones generated by the MonteCarlo procedure. Written when an impulse fitting has been performed (command ^).
<b>LAST0.SLF</b>	Full SLF status vectors file, written at the epoch of the last observation.
<b>LAST0.SLX</b>	SLX status vector file of the optimized body, written at the epoch of the last observation.
<b>LAST1.SLF</b>	Full SLF status vectors file, written at the target epoch reached by the GoToEpoch command.
<b>LAST1.SLX</b>	SLX status vector file of the fitted body, written at the epoch reached by the GoToEpoch command.
<b>MASS.LOG</b>	Log File of the optimized masses in a set of clones generated by the MonteCarlo procedure. Written when the mass of an extra body was included in the fitting.

<b>MILRESID.DAT</b>	Residuals reported by the Astdys/Neodys rwo file (in the case of import of the data), written in the same format as RESID.DAT. Useful for comparison.
<b>NONGRAV.LOG</b>	Log File of the Non-gravitational parameters in a set of clones generated by the MonteCarlo procedure. Written when the parameters were included in the fitting.
<b>ORBIT.LOG</b>	Log File of the successive iterations of the fitting.
<b>POSVEL.LOG</b>	Table of the status vectors of the set of clones generated by the MonteCarlo procedure.
<b>RANDOM.SLX</b>	File of clones of the nominal solution generated by the MonteCarlo procedure, to be imported by <b>Solex</b> .
<b>RANDOMX.SLX</b>	File of clones of the nominal solution for the extra perturbing body, generated by the MonteCarlo procedure, when the elements of an extra body have been fitted.
<b>RESID.DAT</b>	Table of the residuals, rewritten at each iteration.
<b>START0.DAT</b>	Nominal solution at convergence, in the format of a status-vector SLX file.
<b>START0.SLX</b>	Nominal solution at convergence for a fitted perturbing extra body, in the format of status vector.
<b>START1.DAT</b>	Final status vector reached after a GoToEpoch command, in the format of an SLX file.
<b>STARTEL0.DAT</b>	Nominal solution at convergence, in the format of an orbital elements SLX (SLE) file.
<b>STARTEL1.DAT</b>	Final orbital elements reached after a GoToEpoch command, in the format of an SLX (SLE) file.
<b>STARTING.DAT</b>	File of the starting conditions read at the beginning of the first iteration, and written by the <b>S</b> (Set as Start) command after convergence. Written in the format of status vector or orbital elements according to the selection made in the starting menu.
<b>STARTING.SLF</b>	Full file of planets as perturbing bodies. Read in alternative to the library file when the option <b>C</b> (custom File) is activated. This only read, never overwritten by <b>Exorb</b> .
<b>STARTING.SLX</b>	File of the starting conditions of one (or more) external perturbing body to be added to the system. The mass of this perturbing body (the last one if more than one) can be fitted along with the elements of the observed body using the command <b>J</b> . Its orbital elements can also be optimized by the option <b>+</b> (Fit external).
<b>??????X.SLX</b>	File written by the command <b>X</b> (Export SLX) after convergence. Named according to the name of the observations POS file. Same data as in START0.DAT.

**Error Estimates.** The global rms deviation and the standard deviations on the parameters are given as derived from the covariance matrix. The error estimates on the classical elements are derived through a rigorous error-propagation analysis based on the formal standard deviations on the position and velocity elements, their covariances and the numerical derivatives of elements with respect to positions and velocities. A different way of obtaining error estimates (Monte Carlo simulation) is time-consuming but more complete and reliable in that gives the whole probability distribution of the elements. It consists to perform repeated fittings on "synthetic data sets", *i. e.* randomly generated sets of data slightly different from the original observations, and to analyze the statistical distribution of the different results so obtained. This procedure is greatly facilitated by the use of the toggle **\***, which causes a random increment to be given to the positions calculated after the first (number 0) run, with normal distribution having mean value zero and standard deviation sigma (the r.m.s. error resulting from the fitting on the original data). After a suitable number of repetitions (perhaps 50, 100 or even a few thousands if you really want to get a



probability distribution curve for each calculated element) the files POSVEL.LOG and/or CLASSEL.LOG can be further analyzed to obtain the final error estimates.

#### **References and Notes.**

- [1]. Vitagliano, A.: Numerical Integration for the real time production of fundamental ephemerides over a wide time span, *Cel. Mech.* **66** (1997), 293-308.
- [2]. The libraries are the same used by the ephemeris program SOLEX (Release 12.1) and are adjusted to fit the DE431 JPL ephemerides up to a very high precision (see document SOLEX121.DOC).
- [3]. Bate, R. R., Mueller, D. D., White, J. E. *Fundamental of Astrodynamics*, Dover Publ., Inc., New York 1971, pp.122-125.
- [4]. When only angular positions are available as observed data (which is largely the most common case), all positions are converted to orthogonal coordinates normalized to unit distance from Earth, before being used in the optimization procedure.
- [5]. Program by T. S. Kelso, available at <http://www.celestrak.com>