Aldo Vitagliano

ASTRONOMY WITHOUT LOOKING AT THE SKY

A BEGINNER'S VENTURE INTO CELESTIAL MECHANICS

The night sky and the planetary motions have always attracted and fascinated the human mind and this is one of the reasons why among all the sciences, Astronomy earned the unique privilege to be practiced by countless amateurs, who have often produced scientifically significant contributions.

Actually, astronomy enthusiasts belong to different categories, whose mutual boundaries are of course easily crossed. So there are the "esthetes" (visualists and astrophotographers), who just like to observe and reproduce what the night sky offers to their instruments. There are those who are more interested in discovering something new and therefore become photometrists, hunters of comets, asteroids, supernovae and even extrasolar planets. There are finally the "virtuals", who do Astronomy without even watching at the sky, being more interested in the mathematical and computing aspects of this science and often in the rare and curious circumstances of the relative positions of the celestial bodies.

This article reports a personal experience, with the purpose to show how even in the latter category a passion can sprout and grow, up to achieve not only a personal satisfaction, but also some scientifically significant result. In some way, it is the story of the birth and development of the computer program SOLEX, which was initially envisioned as a toy, but over the years evolved up to become a serious and deep investigation tool in the field of Celestial Mechanics.

The seeds.

The beginning of the story dates back more than 50 years, on November 7, 1960, when my dad let me to look at a Mercury transit through the eyepiece of a small telescope. I distinctly remember to have observed it during nearly one hour until the sunset, from the balcony of our apartment on the 4th floor, as the background of the long tree-lined avenue where we lived, which in those days was almost clear of buildings. The peculiar thing was that I was not as much impressed by the phenomenon itself, as by an implicit question: "How could people know in advance that the planet Mercury, which furthermore is invisible most of the time, would be crossing the Sun precisely at that time?". However a 12 years boy, even if asks himself a tricky question, quickly finds a way to forget it. Still, a little seed had entered a concealed turn of my brain, and was hidden there, sleeping.

One decade after that Mercury transits, while I was about to graduate in Chemistry, a second seed happened to sneak close to the first one: a reading from the famous book: "*The Feynman Lectures on Physics*", where at the end of Chapter 9: "*Newton's Laws of Dynamics*", after showing how to solve numerically the motion equations through a slight modification of the Euler method, Feynman writes: "*Thus it take only two minutes to follow Jupiter around the Sun, with all the perturbations of all the planets correct to one part in a billion, by this method*!..." and a little further: "*So, as we said, we began this chapter not knowing how to calculate even the motion of a mass on a spring. Now, armed with the tremendous power of Newton's laws, we can not only calculate such simple motions but also, given only a machine to handle the arithmetic, even the tremendously complex motions of the planets, to as high a degree of precision as we wish!*"

I was very impressed by this reading, and immediately tried to turn it into practice, using some programming notions recently learned in a course, and with the help of a friend who had a job at the Computer Centre of the University. Unfortunately, my test program was likely affected by some bug, because I only got nonsense results. In those days the electronic computers (that one was an IBM 360/40) were as big as a bar bench, and together with their peripherals were occupying a whole large room, as accessible to people as the control room of an Air Force base. Here they were attended by technicians dressed in white lab coats, who looked to me like priests attending the sanctuary of a goddess and who didn't have time to waste for my play games. Being grown up with the logarithm tables and the slide rule, I was fascinated by electronic computers, nevertheless, since I had never been able to learn something without doing it myself from beginning to end, I gave up the numerical experiments to attend full-time to those in the chemical laboratory. And everything went (apparently) into oblivion.

The beginning and the development.

Fifteen more years were gone. Personal computers appeared and spread around, and I had finally the chance to try, fail and retry at my wish, without any control or supervision. Even if I was quite grown up and should be dedicated to my serious professional and familiar duties, I began to use much of my spare time to play like a child and to learn the use of those machines which years before had fascinated me so much, but had been so unreachable. So it happened that, after I got some experience and practice, the seed planted nearly thirty years before sprouted, and I got the idea of build a virtual planetarium for the PC. But how to calculate the planetary positions? Being around lurking inside the Hoepli bookstore in Milan, I was browsing through “Astronomical Formulae for Calculators” by Jean Meeus, and I got shocked! I was a total novice in Astronomy, and to my virgin look, not influenced by the prejudice of a previous knowledge of the subject, those endless trigonometric series of sines and cosines (an example is given in Fig. 1, taken by the successive Italian translation of the book by S. De Meis [*]) appeared as an impracticable solution, not as much for the complication of the formulae, as because they offended my aesthetic sense and my need of understanding.

Saturno

Calcolare v, V, W, ζ, ecc. come per Giove ed anche

Perturbazioni in longitudine media (A)

$+(-0^\circ 814\,181 + 0^\circ 018\,150\,v + 0^\circ 016\,714\,v^2)\sin V$
$+(-0^\circ 010\,497 + 0^\circ 160\,906\,v - 0^\circ 004\,100\,v^2)\cos V$
$+0^\circ 007\,581 \sin 2V$
$-0^\circ 007\,986 \sin W$
$-0^\circ 148\,811 \sin \zeta$
$-0^\circ 040\,786 \sin 2\zeta$
$-0^\circ 015\,208 \sin 3\zeta$
$-0^\circ 006\,339 \sin 4\zeta$
$-0^\circ 006\,244 \sin Q$
$+(0^\circ 008\,931 + 0^\circ 002\,728\,v)\sin \zeta \sin Q$
$-0^\circ 016\,500 \sin 2\zeta \sin Q$

$-0^\circ 005\,775 \sin 3\zeta \sin Q$
$+(0^\circ 081\,344 + 0^\circ 003\,206\,v)\cos \zeta \sin Q$
$+0^\circ 015\,019 \cos 2\zeta \sin Q$
$+(0^\circ 085\,581 + 0^\circ 002\,494\,v)\sin \zeta \cos Q$
$+(0^\circ 025\,328 - 0^\circ 003\,117\,v)\cos \zeta \cos Q$
$+0^\circ 014\,394 \cos 2\zeta \cos Q$
$+0^\circ 006\,319 \cos 3\zeta \cos Q$
$+0^\circ 006\,369 \sin \zeta \sin 2Q$
$+0^\circ 009\,156 \sin 2\zeta \sin 2Q$
$+0^\circ 007\,525 \sin 3\psi \sin 2Q$
$-0^\circ 005\,236 \cos \zeta \cos 2Q$
$-0^\circ 007\,736 \cos 2\zeta \cos 2Q$
$-0^\circ 007\,528 \cos 3\psi \cos 2Q$

Perturbazioni in eccentricità

(I coefficienti sono in unità della settima decimale)

$+(-7927 + 2548\,v - 91\,v^2)\sin V$
$+(13381 + 1226\,v - 253\,v^2)\cos V$
$+(248 - 121\,v)\sin 2V$
$-(305 + 91\,v)\cos 2V$
$+412 \sin 2\zeta$
$+12415 \sin Q$
$+(390 - 617\,v)\sin \zeta \sin Q$
$+(165 - 204\,v)\sin 2\zeta \sin Q$
$+26599 \cos \zeta \sin Q$
$-4687 \cos 2\zeta \sin Q$
$-1870 \cos 3\zeta \sin Q$
$-821 \cos 4\zeta \sin Q$
$-377 \cos 5\zeta \sin Q$
$+497 \cos 2\psi \sin Q$
$+(163 - 611\,v)\cos Q$
$-12696 \sin \zeta \cos Q$
$-4200 \sin 2\zeta \cos Q$
$-1503 \sin 3\zeta \cos Q$
$-619 \sin 4\zeta \cos Q$
$-268 \sin 5\zeta \cos Q$
$-(282 + 1306\,v)\cos \zeta \cos \zeta$
$+(-86 + 230\,v)\cos 2\zeta \cos$
$+461 \sin 2\psi \cos Q$
$-350 \sin 2Q$

$+(2211 - 286\,v)\sin \zeta \sin 2Q$
$-2208 \sin 2\zeta \sin 2Q$
$-568 \sin 3\zeta \sin 2Q$
$-346 \sin 4\zeta \sin 2Q$
$-(2780 + 222\,v)\cos \zeta \sin 2Q$
$+(2022 + 263\,v)\cos 2\zeta \sin 2Q$
$+248 \cos 3\zeta \sin 2Q$
$+242 \sin 3\psi \sin 2Q$
$+467 \cos 3\psi \sin 2Q$
$-490 \cos 2Q$
$-(2842 + 279\,v)\sin \zeta \cos 2Q$
$+(128 + 226\,v)\sin 2\zeta \cos 2Q$
$+224 \sin 3\zeta \cos 2Q$
$+(-1594 + 282\,v)\cos \zeta \cos 2\zeta$
$+(2162 - 207\,v)\cos 2\zeta \cos 2Q$
$+561 \cos 3\zeta \cos 2Q.$
$+343 \cos 4\zeta \cos 2Q$
$+469 \sin 3\psi \cos 2Q$
$-242 \cos 3\psi \cos 2Q$
$-205 \sin \zeta \sin 3Q$
$+262 \sin 3\zeta \sin 3Q$
$+208 \cos \zeta \cos 3Q$
$-271 \cos 3\zeta \cos 3Q$
$-382 \cos 3\zeta \sin 4Q$
$-376 \sin 3\zeta \cos 4Q$

Perturbazioni in perielio (B)

$+(0^\circ 077\,108 + 0^\circ 007\,186\,v - 0^\circ 001\,533\,v^2)\sin V$
$+(0^\circ 045\,803 - 0^\circ 014\,766\,v - 0^\circ 000\,536\,v^2)\cos V$
$-0^\circ 007\,075 \sin \zeta$
$-0^\circ 075\,825 \sin \zeta \sin Q$
$-0^\circ 024\,839 \sin 2\zeta \sin Q$
$-0^\circ 008\,631 \sin 3\zeta \sin Q$
$-0^\circ 072\,586 \cos Q$
$-0^\circ 150\,383 \cos \zeta \cos Q$
$+0^\circ 026\,897 \cos 2\zeta \cos Q$
$-0^\circ 010\,053 \cos 3\zeta \cos Q$
$-(0^\circ 013\,597 + 0^\circ 001\,719\,v)\sin \zeta \sin 2Q$
$+(-0^\circ 007\,742 + 0^\circ 001\,517\,v)\cos \zeta \sin 2Q$
$+(0^\circ 013\,586 - 0^\circ 001\,375\,v)\cos 2\zeta \sin 2Q$
$+(-0^\circ 013\,667 + 0^\circ 001\,239\,v)\sin \zeta \cos 2Q$

$-0^\circ 011\,981 \sin 2\zeta \cos 2Q$
$-(0^\circ 014\,861 + 0^\circ 001\,136\,v)\cos \zeta \cos 2Q$
$-(0^\circ 013\,064 + 0^\circ 001\,628\,v)\cos 2\zeta \cos 2Q$

Perturbazioni in semiasse maggiore

(I coefficienti sono in unità della ses

$-572\,v \sin V$	$+2885 \cos \zeta \cos Q$
$-2933 \cos V$	$+(2172 + 102\,v)\cos 2\zeta \cos Q$
$-33629 \cos \zeta$	$+296 \cos 3\zeta \cos Q$
$-3081 \cos 2\zeta$	$-267 \sin 2\zeta \sin 2Q$
$-1423 \cos 3\zeta$	$-778 \cos \zeta \sin 2Q$
$671 \cos 4\zeta$	$+495 \cos 2\zeta \sin 2Q$
$-320 \cos 5\zeta$	$+250 \cos 3\zeta \sin 2Q$
$-1098 \sin Q$	$-856 \sin \zeta \cos 2Q$
$-2812 \sin \zeta \sin Q$	$+441 \sin 2\zeta \cos 2Q$
$+688 \sin 2\zeta \sin Q$	$+296 \cos 2\zeta \cos 2Q$
$-393 \sin 3\zeta \sin Q$	$+211 \cos 3\zeta \cos 2Q$
$-228 \sin 4\zeta \sin Q$	$-427 \sin \zeta \sin 3Q$
$+2138 \cos \zeta \sin Q$	$-398 \sin 3\zeta \sin 3Q$
$-999 \cos 2\zeta \sin Q$	$+344 \cos \zeta \cos 3Q$
$-642 \cos 3\zeta \sin Q$	$-427 \cos 3\zeta \cos 3Q$
$-325 \cos 4\zeta \sin Q$	*latitudine* eliocentrica le seguenti perturba
$-890 \cos Q$	
$+2206 \sin \zeta \cos Q$	$+0^\circ 000\,747 \cos \zeta \sin Q$
$-1590 \sin 2\zeta \cos Q$	$+0^\circ 001\,069 \cos \zeta \cos Q$
$-647 \sin 3\zeta \cos Q$	$+0^\circ 002\,108 \sin 2\zeta \sin 2Q$
$-344 \sin 4\zeta \cos Q$	$+0^\circ 001\,261 \cos 2\zeta \sin 2Q$

Fig. 1. Main periodic terms for computing Saturn’s orbital elements (from J. Meeus, *Astronomia con il Computer*, Hoepli 1990)

Today I understand and can even appreciate those series, to the point that sometimes I built a few of them just for the fun of it, but in those days they appeared to me both obscure and terribly ugly. Hence, prompted by my contempt of those horrible formulas , thinking and rethinking on the matter the second hidden seed sprout off: why not start from the first principles, that is from the Newton gravitation law ? The matter is to know the mass of each planet, and to know its position and velocity at a given instant. By computing the acceleration experienced by each body through the Newton’s law, it becomes simple to infer its new position and velocity one moment later, and so to build up step-by-step its path, so long as desired, taking in account *ipso facto* all the acting forces and hence all the perturbations. It is only matter of repeating the same simple calculations *ad infinitum*, and what are computers made for if not to this kind of job? This is exactly the principle of the **numerical integration** of the orbits, so well outlined by Feynman, and whose essential features are summarized in Fig. 2.

The gravitation law …

Isaac Newton

$$\vec{a}_{ki} = -\frac{Gm_i}{r^2}\frac{\vec{r}}{|r|}$$

Acceleration of the k_{th} body by the mass of the i_{th} body, at a distance r, written as a vector. The total acceleration on body k is the sum of all the contributions by the different masses m_i.

and a basis for the numerical integration of planetary motions

Leonhard Euler

$$\Delta x = v_{x0}\Delta t + \frac{1}{2}a_{x0}(\Delta t)^2$$

$$x_1 = x_0 + \Delta x$$

$$\Delta v_x = \frac{(a_{x0} + a_{x1})}{2}\Delta t$$

$$v_{x1} = v_x + \Delta v_x$$

Computing the x coordinate and v_x velocity after a time interval Δt starting from the initial values x_0 e v_{x0}, with the modified Euler method (midpoint method). The components y and z are calculated similarly.

Fig. 2

Hence, as Feynman wrote, knowing the initial positions and velocities of all the planets (the "**initial conditions**") , the elementary arithmetics would be enough to compute their past and future orbits to any desired precision for as long as we wish … Well, not really, because the midpoint method shown in Fig. 1 is very simple, but it is as well not terribly accurate. Since the acceleration changes with the position, the formulas in Fig 2 give an approximate value of the coordinates and velocities (the **status vectors**) at the end of the interval, with an error growing with the square of the stepsize **Δt**. It is in principle possible to use such a small stepsize to make the error negligible, but this choice would imply too many steps and the method won't be very convenient. In practice, for the cases requiring an extreme precision (as the orbit calculations), it is necessary to use more sophisticated procedures. After an endless number of experiments, I finally landed on a so-called *extrapolation method*, which combines an extreme efficiency with a fair conceptual simplicity and easiness to code in a program (both important in my case). In this method a modification of the midpoint method is applied several times to the same big interval **T**, which is "scanned" each time using a stepsize **Δt** corresponding to **T**, **T/2**, **T/3**, **T/4** … and so on, and the sequence of temporary results is extrapolated to **Δt**=0, the mythical golden condition that would in theory produce the exact solution. By this way the final error is very small, and moreover it decreases by reducing the stepsize according to a power of **T** which is the double of the number of successive "scans" made of the interval **T**. Most conveniently, **T** is divided in 1, 2, 3, 4, 5, 6, 8, 10 substeps, resulting in 8 successive scans and an error that is reduced by a factor of 2^{16} ~ 65000 (!) by just halving the stepsize **T and** doubling the computing time .

All this looks pretty simple, but how to get sufficiently accurate and precise starting positions and velocities? Moreover, how to make sure that the positions computed thereafter by the numerical integration are actually the correct ones?

In those days (around year 1990) the best available data were offered by the *Floppy Almanac* or by the ***I**nteractive **C**omputer **E**phemeris* (**ICE**) issued by the U.S. Naval Observatory, who however provided the positions but not the velocities. So what to do? The answer was in a golden old booklet that I happened to find in a shelf of the Hoepli bookstore in Milan, *Fundamentals of Astrodynamics*, by Bate, Mueller and White (Dover Publication, Inc., N.Y.). Jumping over pages and pages of not very friendly formulas, in the second chapter I found both the idea and the suitable method: why not using
The **ICE** ephemerides as they were true observations and, starting from tentative positions, apply on them both the numerical integration and the Gauss least-squares method, in order to get the correct starting

conditions for all the planets and the Moon? By this way not only the initial conditions are obtained, but also the measure of how much the computed orbits deviate from the reference ones, taken as “true”. All this by following the precept of Gauss, stated more than 200 years ago in his *Theoria Motus Corporum Coelestium*:

> *Systema itaque maxime probabile valorum incognitarum* p, q, r, s etc. *id erit, in quo quadrata differentiarum inter functionum* V, V′, V″ etc. *valores obseruatos et computatos summam minimam efficiunt*, siquidem in omnibus obseruationibus idem praecisionis gradus praesumendus est.

“Therefore, that will be the most probable system of values of the unknown quantities p, q, r, s, etc., in which the sum of the squares of the differences between the observed and computed values of the functions V, V, V", etc. is a minimum, if the same degree of accuracy is to be presumed in all the observations.”

However, after quite a bit of time devoted to programming, testing and debugging, an unpleasant revelation arrived: with the Newton’s law alone, things were not as simple as I thought! After optimizing the initial conditions, the external planet do agree quite well with the reference data, but Mercury and the Moon refuse to do that, and their deviation from the reference data intolerably increases with time (Fig. 3).

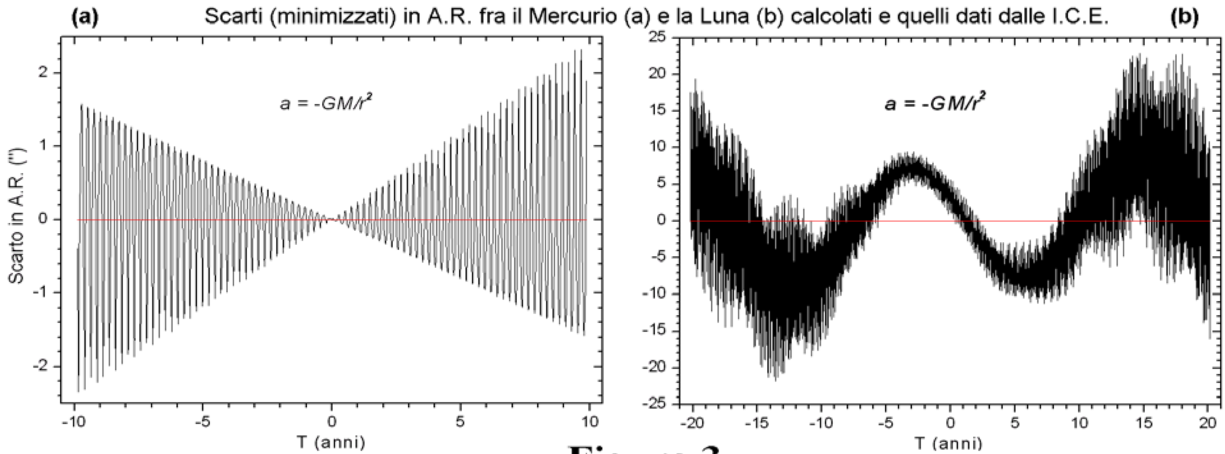


Figura 3

Fig. 3. Deviations (arcsecs) in the Right Ascension of Mercury (a) and the Moon (b) after optimization of a purely Newtonian force model (time in years).

Well, even if I was a novice of celestial mechanics, I should have expected that: the purely Newtonian model is not adequate to fully account for the orbital motion, or at least is not adequate in the case of Mercury and the Moon. It didn’t take long to discover the dynamic model of JPL, which adds to the motion laws the so-called relativistic PPN (Parametrized Post Newtonian) corrections (Fig. 4, formula taken from *Explanatory Supplement to the Astronomical Almanac*, 1992).

$$
\begin{aligned}
\ddot{\mathbf{r}}_{i\,\text{point mass}} = \sum_{j\neq i} \frac{\mu_j(\mathbf{r}_j-\mathbf{r}_i)}{r_{ij}^3}\Bigg\{ 1 - \frac{2(\beta+\gamma)}{c^2}\sum_{k\neq i}\frac{\mu_k}{r_{ik}} - \frac{(2\beta-1)}{c^2}\sum_{k\neq j}\frac{\mu_k}{r_{jk}} + \gamma\left(\frac{v_i}{c}\right)^2 \\
+ (1+\gamma)\left(\frac{v_j}{c}\right)^2 - \frac{2(1+\gamma)}{c^2}\dot{\mathbf{r}}_i\cdot\dot{\mathbf{r}}_j - \frac{3}{2c^2}\left[\frac{(\mathbf{r}_i-\mathbf{r}_j)\cdot\dot{\mathbf{r}}_j}{r_{ij}}\right]^2 + \frac{1}{2c^2}(\mathbf{r}_j-\mathbf{r}_i)\cdot\ddot{\mathbf{r}}_j\Bigg\} \\
+ \frac{1}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}^3}\left\{[\mathbf{r}_i-\mathbf{r}_j]\cdot[(2+2\gamma)\dot{\mathbf{r}}_i-(1+2\gamma)\dot{\mathbf{r}}_j]\right\}(\dot{\mathbf{r}}_i-\dot{\mathbf{r}}_j) \\
+ \frac{3+4\gamma}{2c^2}\sum_{j\neq i}\frac{\mu_j\ddot{\mathbf{r}}_j}{r_{ij}} + \sum_{m=1}^{5}\frac{\mu_m(\mathbf{r}_m-\mathbf{r}_i)}{r_{im}^3}, \qquad (5.211\text{–}1)
\end{aligned}
$$

Fig. 4. The equations of the JPL dynamic planetary model.

From my point of view, this frightening expression was unacceptable for computing each individual acceleration, partly because of its complication (my appreciation for things goes with the square of their simplicity), but mostly because of its heavy extra computing load, which would take about 98% of machine time. So I was wandering for any effective shortcut to the problem, and, remembering something I had read a few years before about the relativistic advance of the perihelion of Mercury, I experimented some tentative simple corrections to the Newton's law. Indeed, the shortcut was there, even too simple to believe.

A small correction to the Newton's law is enough to put Mercury on the right track, lowering the maximum discrepancy from the reference data to less than a milliarcsec and, most importantly, making it stable and not growing with time. (Fig. 5a). To put the Moon on the right track was rather more difficult, because not only some relativistic correction is needed, but especially because both the Earth and the Moon are not spherically symmetric and, over distances not exceedingly larger than their respective sizes, cannot be modeled as pointless masses, to not speak of the transverse acceleration due to tidal effects. However, part of the corrections to comply with the lunar JPL model could be made in a rigorous way, and for the rest I could find some acceptable shortcut, producing a valid and time-stable solution (Fig. 5b).

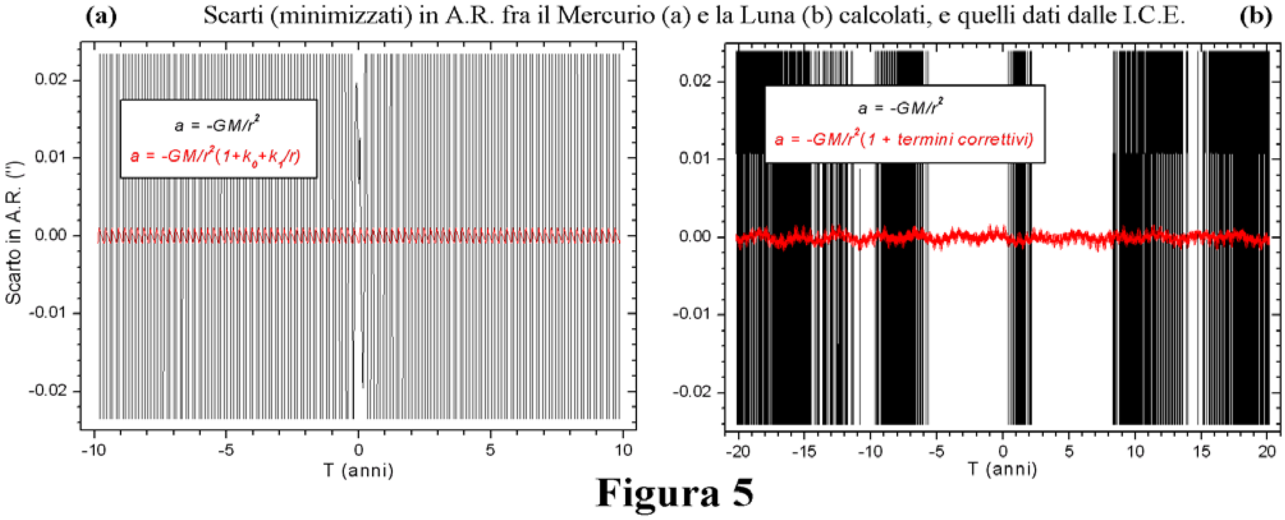


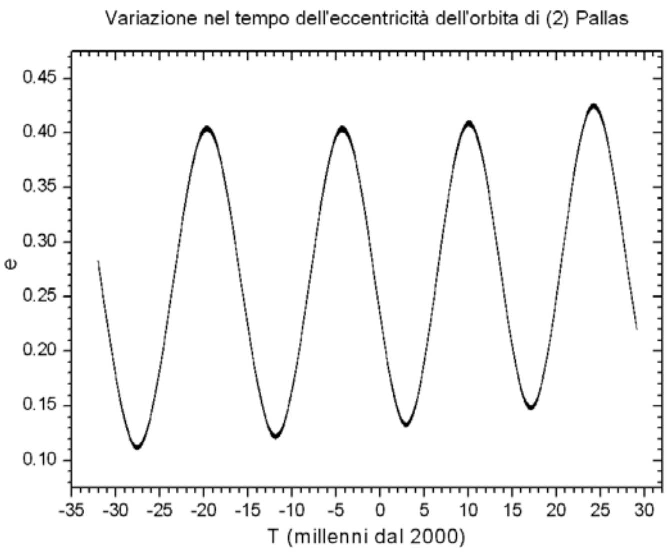
Fig. 5. **Deviations (arcsecs) in the Right Ascension of Mercury (a) and the Moon (b) before (black) and after (red) adding the corrective terms to the purely Newtonian model (time in years).**

After the past, the present.

So, the engine was built and, most importantly, it was working well. It also appeared to be an original construction, and therefore, being used to the writing of scientific papers, I submitted a manuscript to *Celestial Mechanics & Dynamical Astronomy*, and the paper was published. It might seem that the most was done. Not at all! For a software to be even slightly useful to somebody, it is necessary to assemble a friendly user interface, input and output routines, the various commands, the graphics, and all the accesory functions. If the original idea of a Planetarium has to be kept, stars should be displayed in a reasonably realistic way and animations should be possible. All these things required much more time and efforts than the solution of the main problem, much in the same way as in a new house all the flooring, the fixtures and the furniture cost more than the framework, although only a well made and solid framework can guarantee a long-lasting quality. But let me skip one or two decades of virtual craftmanship and jump straight to the present, consisting in a Windows software package made of two programs: **Solex** and **Exorb** (Fig. 6).

Fig. 6. The opening screens of the two programs Solex ed Exorb.

They are included in the same package and can exchange datafiles between each other. The former is aimed to compute planetary orbits and positions, and also includes a graphic planetarium. The latter is aimed to compute orbital elements from astrometric observations. They are not much scenic and they don't do things that many other programs do (for example they don't handle planetary satellites nor planetary physical ephemerides or Saturn's rings, and they cannot be interfaced with a telescope). Perhaps they look old-fashioned and not too friendly to the occasional user, but on the other hand, they do things that no other software does… I'm not describing here the many functions of **Solex**, that you might enjoy to explore by yourself, and I just want to list a few of its special tasks, which are made possible by the numerical integration approach. **1.** Handling of strongly perturbed objects (comets and asteroids) over moderately long time intervals, including the corresponding error estimates. **2.** Search of astronomical curiosities over a long time interval (one example is the simultaneous transit of Venus, Earth, and Moon, as seen from Mars, which took place on Julian year -254328 and shown in the opening screen of the program). **3.** Determination of the medium and long term evolution of planetary and asteroidal orbits (Figs. 7 and 8). **4.** Display of orbital dynamics (see the animations page on the Solex website). **5.** Search for conditions of minimum or maximum distances (spatial or angular) between solar system bodies. This functionality, combined with the features of the companion program **Exorb**, led in the past to discover an exceptionally close approach between a small asteroid and **(15) Eunomia**, which in turn allowed to determine the mass of the latter to a previously unmatched precision. But since I mentioned **Exorb**, and just to give to it its share of credit, let me tell another story, halfway between past and present...



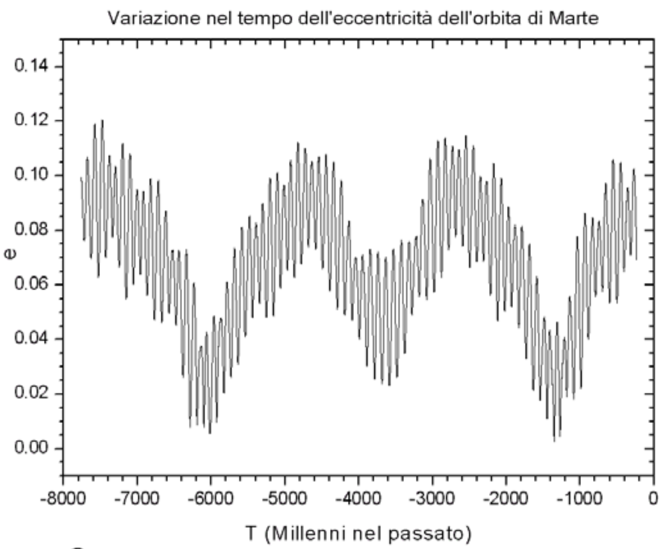


Figura 8

Fig.7. The secular variation of the eccentricity of Pallas' orbit.
Fig 8. Long time variation of the eccentricity of Mars.

In the middle of 19th century, celestial mechanics was a frontier science, and one of its most successful achievements surely was the prediction of Neptune made by Urbain Leverrier. In 1845 Leverrier was a brilliant young theoretician working at the Paris' Observatory, and was assigned by his director François Arago the hard task of solving a problem which caused many troubles among the astronomers of the time. The problem resided in the oddness of Uranus' orbit, whose observations were in agreement with the current theory (including perturbations by Jupiter and Saturn) only within a limited time interval, leaving out beyong any reasonable doubt both the old observations and the most recent ones (Fig. 9a). Fig. 9b shows how the 1822 heliocentric conjunction did heavily affect the discrepancies between the theory and the observations made thereafter.

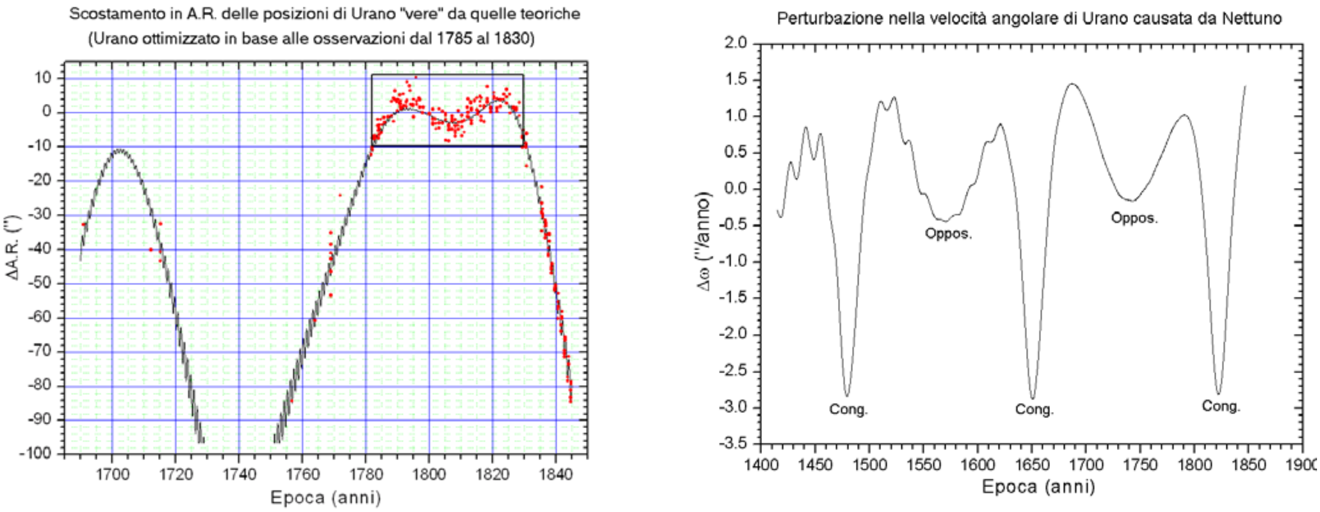


Fig. 9. (a) Difference in RA between the true positions of Uranus and those predicted by the theory. The red dots represent the actual observations, the black curve represents the true positions and the zero corressponds to the theoretical positions at the epoch. (b) The influence of Neptune on the computed angular velocity of Uranus, expressed as a difference ("with Neptune" minus "without Nepune". The maximum slowing down takes place at the heliocentric conjunctions.

Leverrier took immediately the right approach, together with another young mathematician, John Couch Adams at Greenwich. Both worked on the hypothesis that the anomalies in Uranus' orbit were due to the presence of an eighth planet not perturbing Saturn, and therefore situated outside the orbit of Uranus. Both mathematicians found that the hypothesis made the resulting theory to agree with the observations, with the only exception of an old one, made by Flamsteed in 1690. Both predicted the mass of the new planet, devising a likely orbit and predicting what should be its celestial position at a given epoch (Fig. 10). However Leverrier, besides being more accurate in his impressive disentanglement of the problem, involving one year of work and more than ten thousand pages of calculations, was also more lucky. He was working in complete autonomy, while the younger Adams was under the control of an oppressive boss (Airy). The result was that the goal was first achieved by Leverrier, who in 1846 published three papers both in "*Comptes Rendues de l'Academies des Sciences*" and "*Astronomische Nachrichten*", and in september 1846 sent a letter to Gottfried Galle in Berlin, indicating the position where to look for the eighth planet. Galle got the letter on September 23, and the very same night started to explore the suggested region of the sky, with the help of the young astronomer d'Arrest who was browsing through a brand-new star catalog. At midnight he found a 8th magnitude star that was not listed in the catalog, at less than 1 degree from the most likely position suggested by Leverrier, and the next night the discovery was confirmed: the "star" had moved: it was the eighth planet predicted by Leverrier!

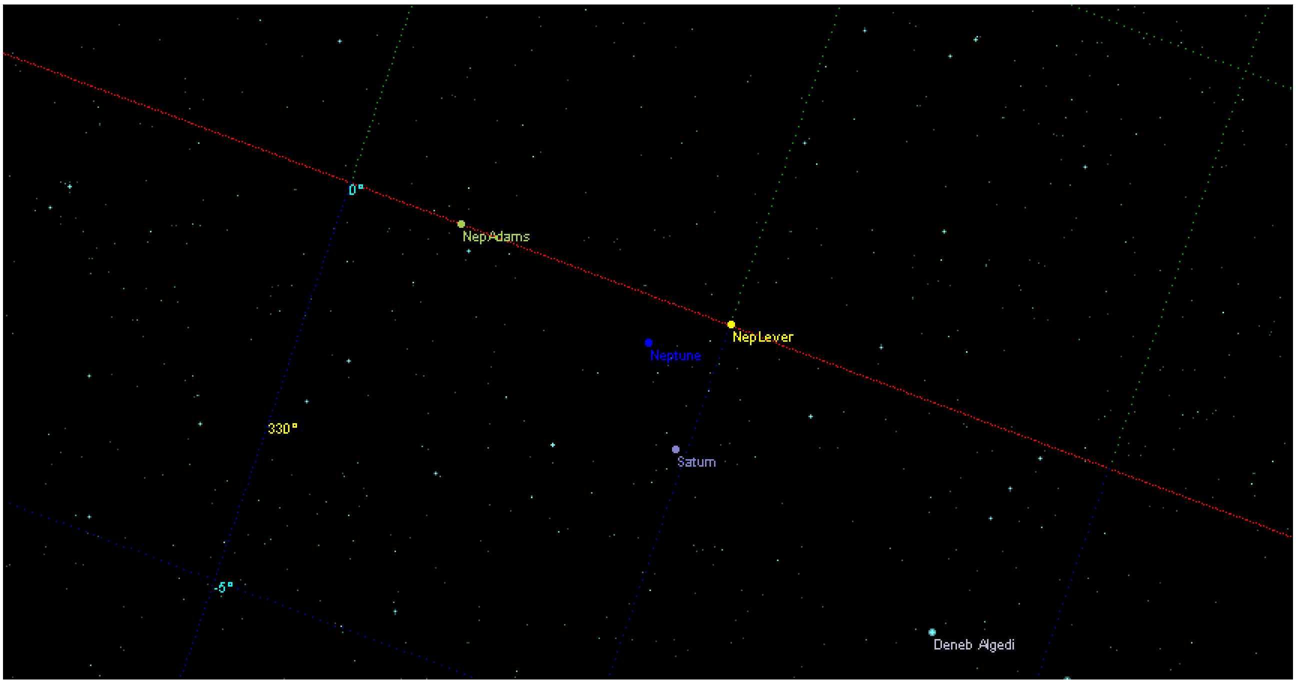


Fig. 10. Observed and predicted positions of Neptune at the epoch of its discovery. The red dotted line represents the ecliptic.

Hovever, the observations taken in the following months led to conclude that the orbit of the eighth planet was quite different from those suggested by Leverrier and Adams (Fig. 11), to the point that the american mathematician Benjamin Peirce (perhaps being a bit jealous of the european discovery) remarked that the discovery was more an "happy accident" than the result of an accurate analysis of the motion of Uranus. To what extent is this true ?

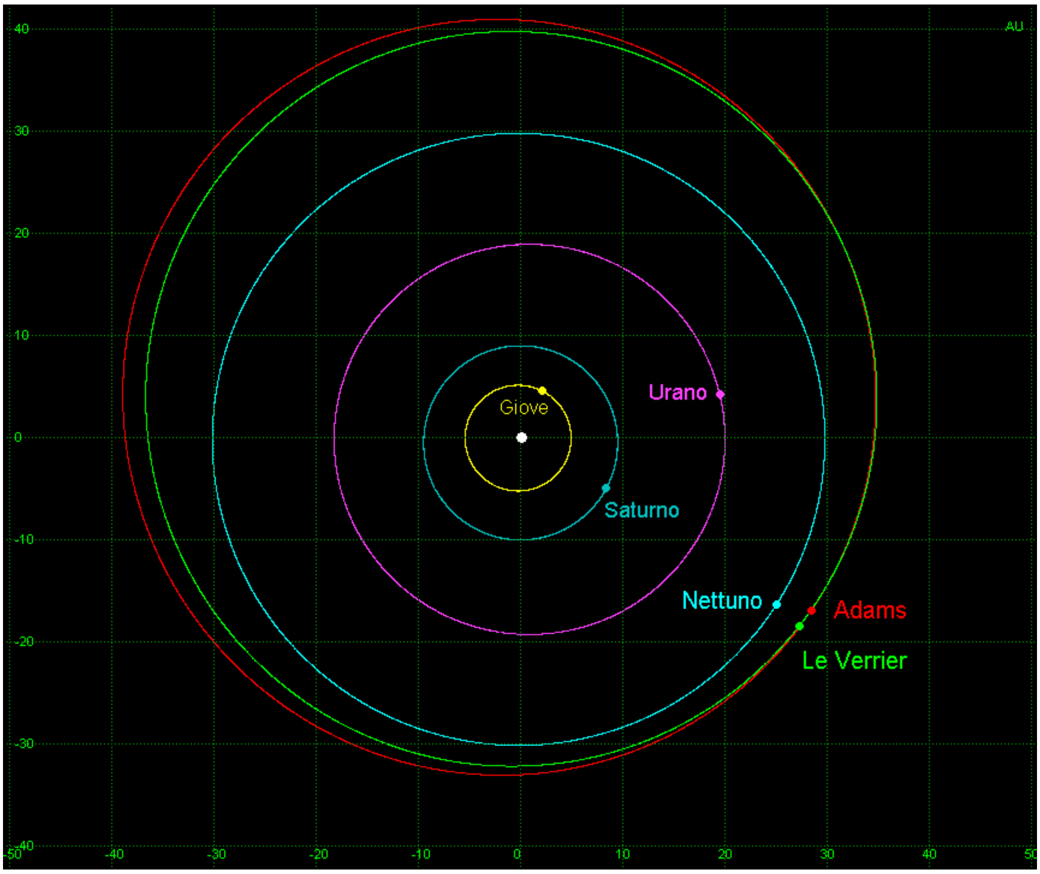
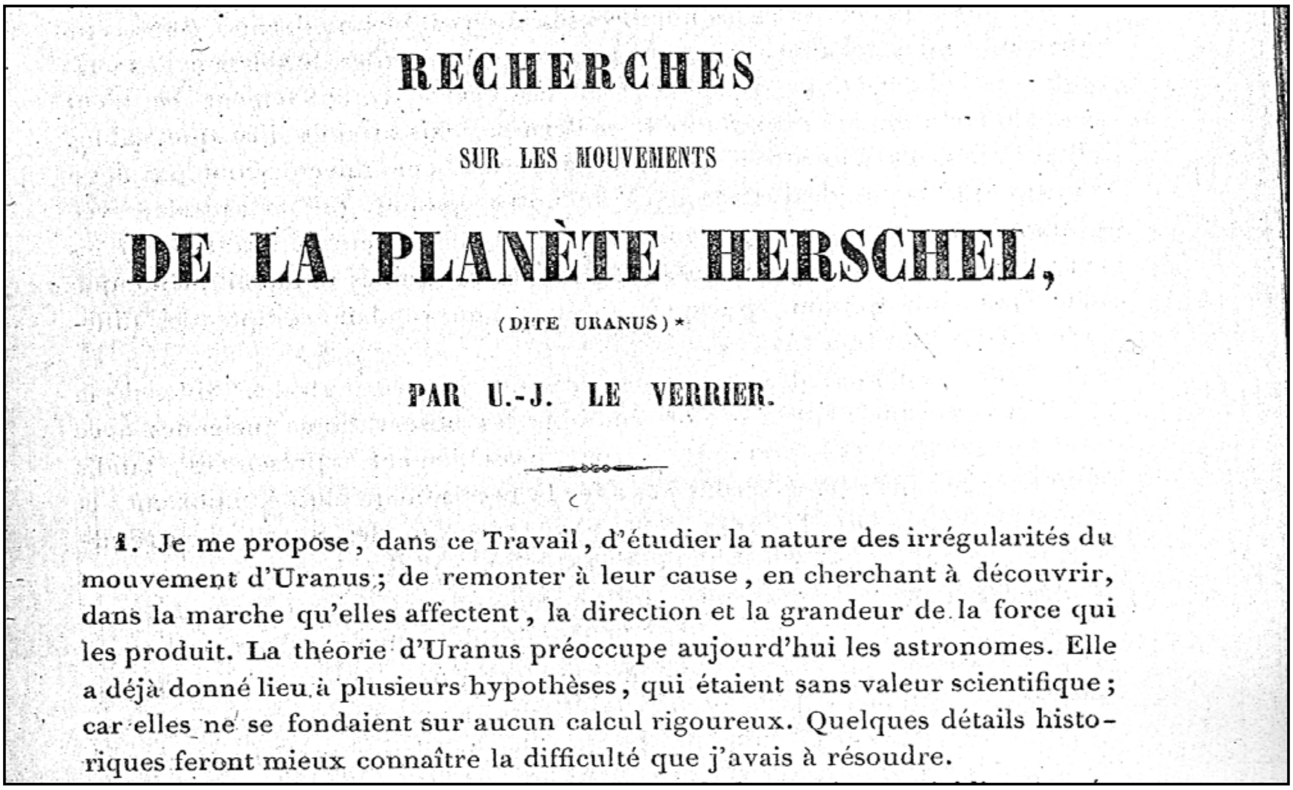


Fig. 11. The "wrong" orbits predicted by Adams and Leverrier, compared with the true one.

Under these premises, since the appropriate tools were in my hands already, how could I resist to the temptation to answer the above question personally and directly? Hence I added to **Exorb** a functionality which is probably a quite rare peculiarity: the ability to search and optimize, starting from the observations of a solar system body, not only its orbital elements but also those of a further perturbing body, including its mass. In other words, the ability to repeat the Leverrier's work without making his approximations, using the brute computing force of a few billions of floating point operations per second… So I patiently copied to a file all the 260 observations of Uranus used by Leverrier and published, thanks to his carefulness, in his majestic work "*Recherches sur les mouvements de la planète Herschel, dit Uranus*" (Fig. 12) and I started to work on them, or, better, I let the electronic microcircuits work on them.



RECHERCHES

SUR LES MOUVEMENTS

DE LA PLANÈTE HERSCHEL,

(DITE URANUS) *

PAR U.-J. LE VERRIER.

1. Je me propose, dans ce Travail, d'étudier la nature des irrégularités du mouvement d'Uranus; de remonter à leur cause, en cherchant à découvrir, dans la marche qu'elles affectent, la direction et la grandeur de la force qui les produit. La théorie d'Uranus préoccupe aujourd'hui les astronomes. Elle a déjà donné lieu à plusieurs hypothèses, qui étaient sans valeur scientifique; car elles ne se fondaient sur aucun calcul rigoureux. Quelques détails historiques feront mieux connaître la difficulté que j'avais à résoudre.

Fig. 12. The first page of the huge paper by Leverrier, published in *Connaissance des Temps pour 1849*

And the result? By adopting the semplification used by both Adams and Leverrier of constraining Neptune on the ecliptic plane, a whole family of orbits is found, all of them reducing the discrepancies between the observed and calculated positions of Uranus within acceptable values, totally compatible with the instrumental errors of those times. The discrepancies (residuals) do not change much within the family of orbital solutions, while the predicted mass of the perturbing planet progressively increases with the size of the orbit. Fig 13 shows 13 of those solutions, together with the orbits calculated by Adams and by Leverrier. Their semimajor axis increases from 28 to 40 AU, and the optimal solution is of course given by the true Neptune, while among those in the ecliptic plane the best one is n. 3 (a = 30 AU), very close in projection to the true one.

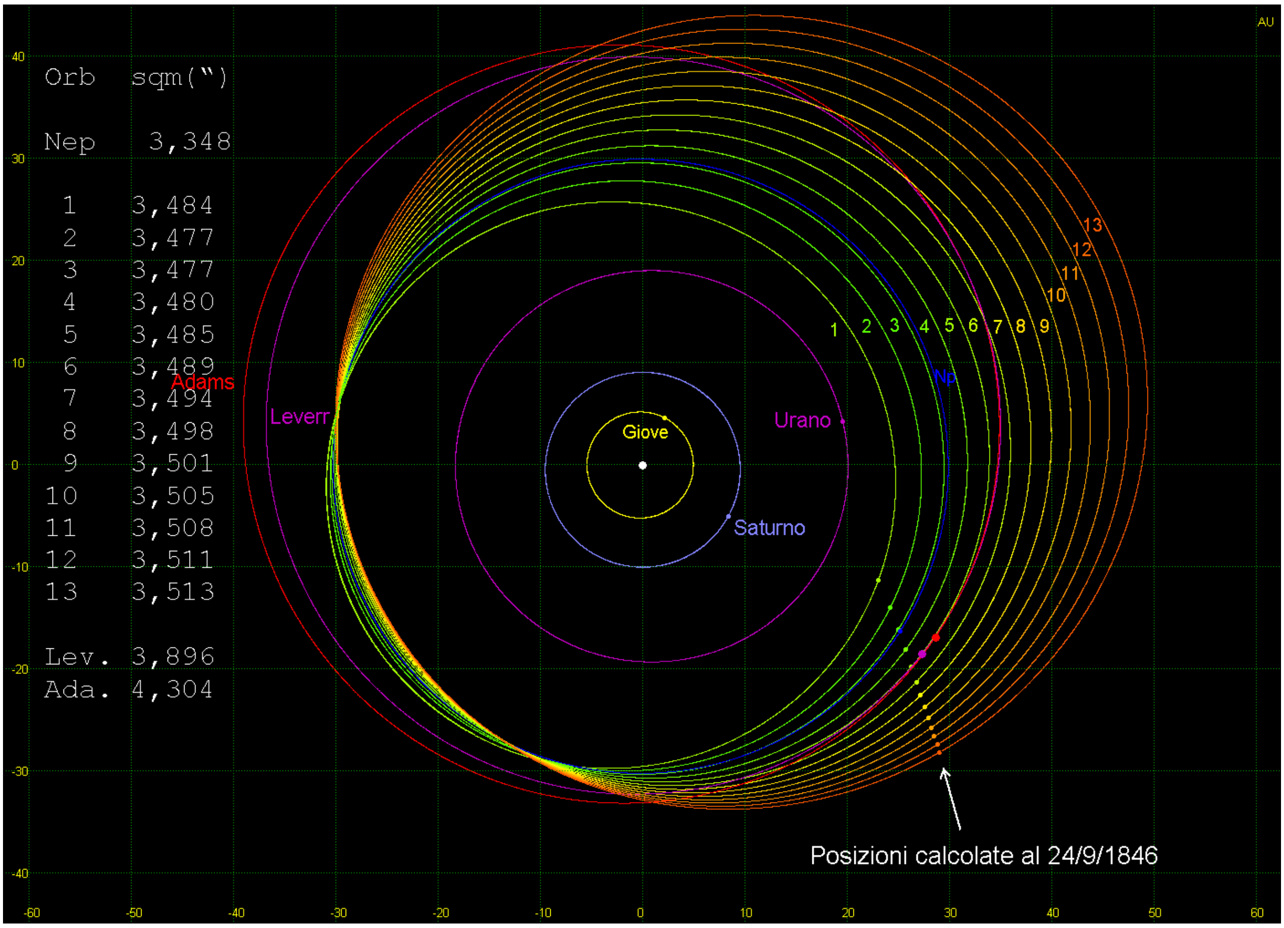


Fig. 13. The family of orbits determined by Exorb for an hypothetical Neptune, using the same observations available to Leverrier in 1845, and the respective mean-squared residuals of the Uranus' observations. The Adams and Leverrier solutions and their mean-squared residuals are also shown.

A still unanswered question is why both Leverrier and Adams found solutions which definitely do not belong to the family of the best ones, and why they could not make their theory to agree with the oldest observation, made by Flamsteed in 1690, which according to the modern results is not even among the worse ones. To answer this question it would be necessary to understand the details of the method used by Leverrier and to repeat his calculations skipping all the approximations he possibly adopted. This is far beyond my abilities, and therefore I only observe that, compared with discrepancies larger than one arcminute that appear when a perturbing planet is not taken in account, even their solutions are acceptable, having mean square residuals not much larger than modern solutions. It is also worth noting that all the solutions place Neptune within a limited arc of the ecliptic, during the crucial years of the most intense perturbations (1800-1845), so that the possible error in the calculated position of Neptune at the epoch of its discovery (September 24, 1846) does not exceed 10° (Fig.14)

In conclusion, going back to the initial question of assessing whether or not Benjamin Peirce was right in Describing the discovery of Neptune as an "*happy accident*", I would say that he was borderline between being right and wrong. With the tools available in the middle of the 19th century, and for nearly a century thereafter, nobody could have done much more, except perhaps to bet on a circular orbit and therefore strongly reduce the complication of the problem. The skill, shared by Adams, was to predict the existence of Neptune in the correct region of the sky, the good luck was to locate it at less than 1° from the true position rather than 10° away from it. But in all big discoveries, good luck is part of the game!

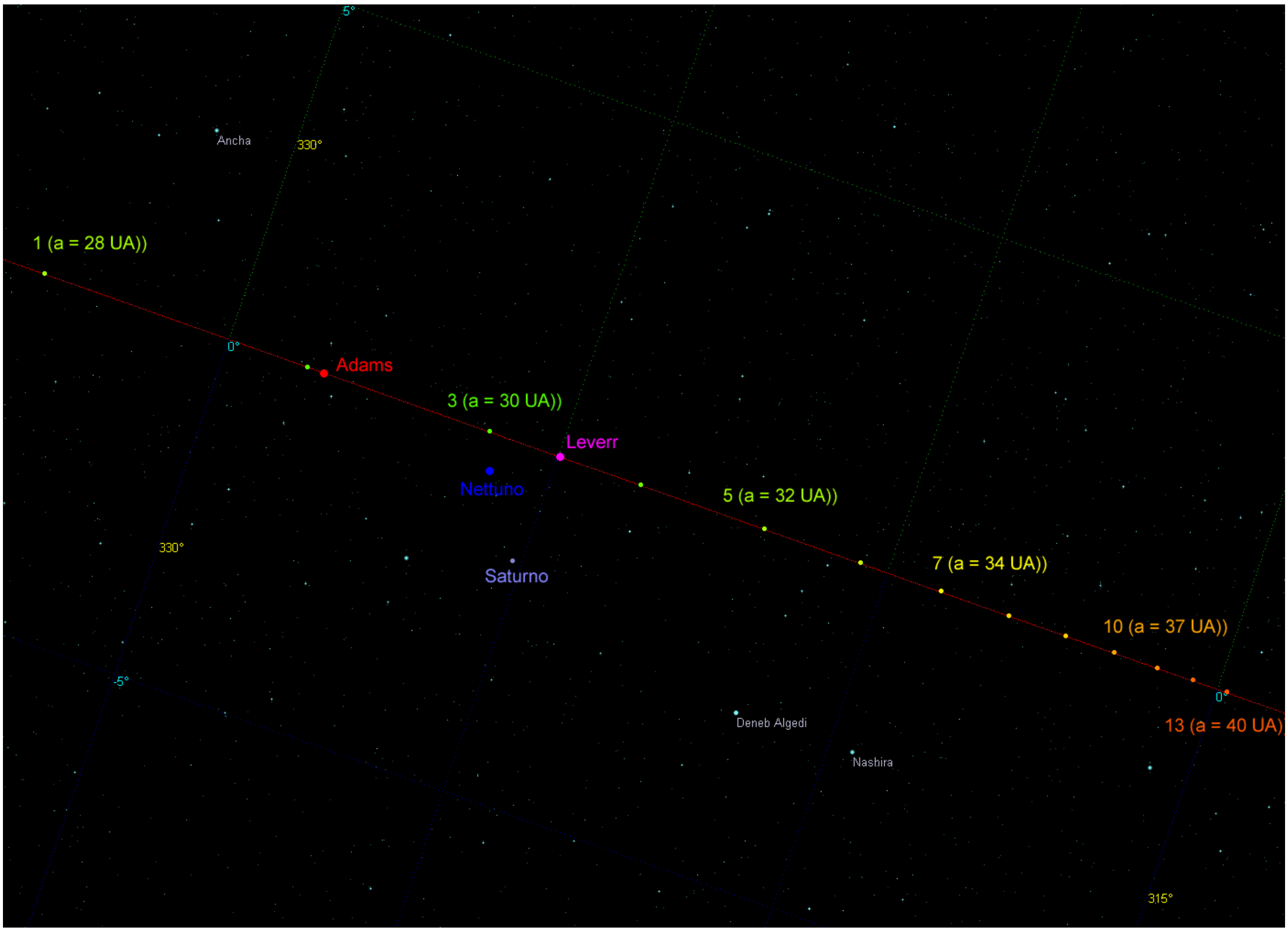


Figura 14. The positions of Neptune at the date of its discovery, as predicted by all the solutions.