Making and using clones *by Aldo Vitagliano, 2018,10,15.*

When **Exorb** determines the orbital elements of a body, the solution gives the most likely values of the elements (those minimizing the sum of the squares of the residuals, assuming that the data are free from systematic errors) along with their 1-σ uncertainty. This is the so-called "nominal solution", which is as much close to the true one as smaller are the σ.

If we want to predict where the body was (or will be) at a given epoch in the past (or in the future) we will also want to know what is the likely error of this prediction, because a prediction without an associated probable error is of little use.

To make any wanted prediction is an easy matter, because by importing the elements into **Solex** we can propagate the orbit as far in time as we like, but the error estimate is another question, because we don't know how the initial errors propagate with the evolution of the orbit.

Here comes the use of the "**clones**", which can be generated by **Exorb** through two different procedures.

Clones are random "virtual bodies" (each represented by a set of orbital elements) such that, if you take a large number of them, their distribution around the nominal solution matches the probability distribution of the deviations of the elements from their most likely (nominal) values.

Therefore, rather than considering just the nominal solution, we can import into Solex a whole cloud of, say, 100, 1000 or 10000 clones, and from that we will obtain not only an average prediction of the orbital evolution but also a reliable estimate of the error propagation with time.

Once the nominal solution has been found, **Exorb** can produce any number of clones by either of two different procedure, which are essentially equivalent.

The first one (command [**K**]) is usually more convenient because of its immediacy. Making good use of the covariance matrix resulting from the fitting procedure, it generates a file named **CLONES.SLX**, which can be imported by **Solex** to investigate the path and the spread of the propagated orbits.

The second procedure (command [*] in the main menu) should in principle be more reliable, but it takes a long computation time, because it needs to perform a new fitting for each generated clone. It is based on the MonteCarlo generation of "synthetic" sets of observations, each set submitted to the fitting procedure and thereby producing one clone. A synthetic set of observations is constructed from the positions calculated by the nominal solution at the times of each actual observations. To these calculates positions a random deviation is applied having the same σ as the assumed σ of the actual observation. Therefore a "synthetic" set of observations represents a set of observations that although were not actually made, **could** have been made at the same times in absence of systematic errors. The clones generated by the MonteCarlo procedure are written by **Exorb** to a file named **RANDOM.SLX**.

The reason to have two different almost equivalent but independent procedures for generating clones is to give the possibility to cross-check the reliability of the results by comparing the two different outputs.

Let's now look at some more information embedded in a cloud of clones, other than its shape and size.

Not all the clones are equally likely. Roughly speaking, those which deviate less from the nominal solution are more likely than those that deviate more. In other words, when a large number of clones is generated, their cloud will be more dense in proximity of the nominal solution and progressively less dense as they elements depart more and more from those of the nominal solution.

How much a given clone departs from the nominal solution (in a way an estimate of its likelihood) is measured by its $\Delta\chi^2$, a normalized quantity that can be computed from the set of deviations and a matrix resulting from the fitting procedure. The average value of $\Delta\chi^2$ is equal to the number of fitted parameters (usually 6 for the orbital elements, but can be higher when more parameters are determined).

The quantity $\Delta\chi^2$ is then related to the likelihood region of the corresponding clone through the "Chi square distribution" whose tables can be found in most mathematical handbooks and are widely spread in Internet. Below I transcribe an abstract for the case of 6 determined parameters (6 degrees of freedom), in order to explain the meaning of this relation or, in other words, how to read such tables.

% Confidence	99.9	99.5	99	97.5	95	90	**75**	50	25
$\Delta\chi^2$	22.5	18.5	16.8	14.4	12.6	10.6	**7.84**	5.35	3.45

A single example will clarify how to read such tables. Let's take the figures stressed in bold: their meaning is that if the $\Delta\chi^2$ value is lower than 7.84 the corresponding clone lies within a confidence region of 75%, or, conversely, that there is a 25% chance that some clone with a higher $\Delta\chi^2$ value be representative of the true body (the true set of elements).

When clones are generated by **Exorb**, the corresponding $\Delta\chi^2$ are written to a file named **DChi2.LOG** (if they come from the first procedure) or **DChi2R.LOG** (if they come from the MonteCarlo procedure). In addition they are also embedded in the color code of the corresponding record of the generated **SLX** file, provided that the default option and not a custom fixed color is selected in the EXORB.CFG file.

The default color of the nominal solution ($\Delta\chi^2$ value = 0) is pure green (009900 in the BBGGRR codification adopted by **Solex**). To this an amount of red is added as the $\Delta\chi^2$ value increases, up to reach pure yellow (009999 in the BBGGRR codification adopted by **Solex**) when the $\Delta\chi^2$ value is equal to its expected average (equal to the number of fitted parameters, normally 6). After that and up to a $\Delta\chi^2$ value equal to twice the average, the green component is removed progressively, up to reach pure red (000099 in the BBGGRR codification adopted by **Solex**). After that and up to a $\Delta\chi^2$ value equal to three times the average, an amount of blue is added progressively, up to reach pure magenta (990099 in the BBGGRR codification adopted by **Solex**). Finally, with $\Delta\chi^2$ values larger than three times the average, the red component is removed progressively, up to reach pure blue (990000 in the BBGGRR codification adopted by **Solex**) when $\Delta\chi^2$ is four time (and beyond) its expected average value.
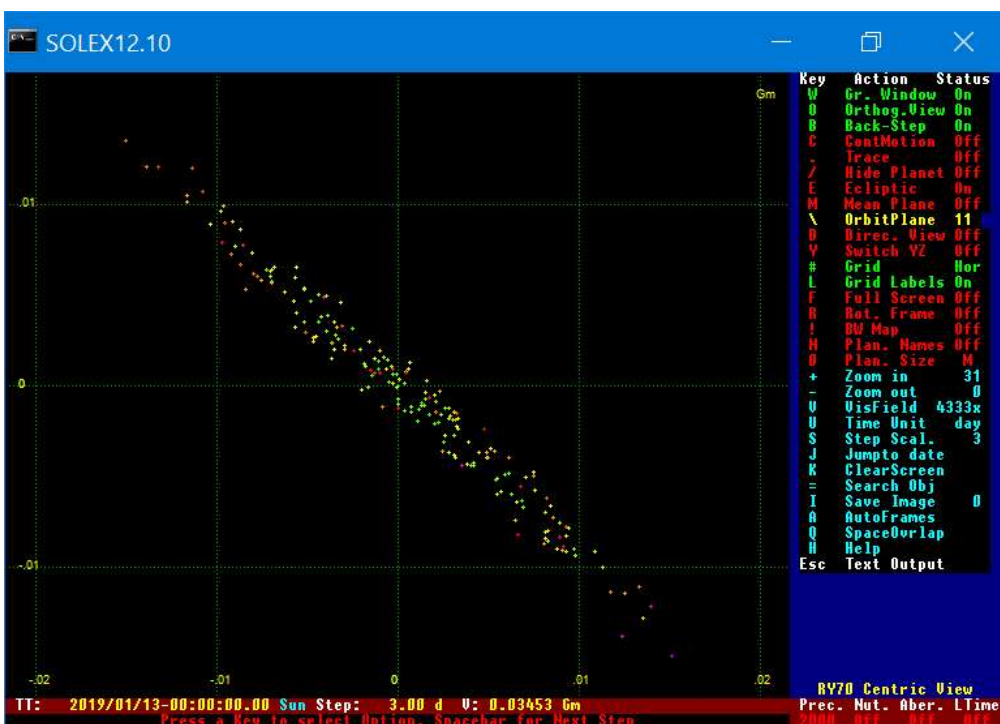
The figure shows a cloud of 200 clones of the NEO 2018RY7, at the epoch 2019, Jan 13, derived from 55 observations available on October 13, 2018. The cloud is centered on the nominal solution and projected on the orbital plane of the body. The extent of the cloud shows that the positional uncertainty is of the order of 0.01 Gm (10000 km).